

A characterisation of isometries with respect to the Lévy-Prokhorov metric

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Abstract. According to the fundamental work of Yu.V. Prokhorov, the general theory of stochastic processes can be regarded as the theory of probability measures in complete separable metric spaces. Since stochastic processes depending upon a continuous parameter are basically probability measures on certain subspaces of the space of all functions of a real variable, a particularly important case of this theory is when the underlying metric space has a linear structure. Prokhorov also provided a concrete metrisation of the topology of weak convergence today known as the Lévy-Prokhorov distance. Motivated by these facts, the famous Banach-Stone theorem, and some recent works related to characterisations of onto isometries of spaces of Borel probability measures, here we give a complete description of surjective isometries with respect to the Lévy-Prokhorov metric in case when the underlying metric space is a separable Banach space. Our result can be considered as a generalisation of L. Molnár's earlier Banach-Stone-type result which characterises onto isometries of the space of all probability distribution functions on the real line with respect to the Lévy distance. However, the present more general setting requires the development of an essentially new technique.

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