KAM, α -Gevrey regularity and the α -Bruno-Rüssmann condition

ABED BOUNEMOURA AND JACQUES FÉJOZ

Abstract. We prove a new invariant torus theorem, for α -Gevrey smooth Hamiltonian systems, under an arithmetic assumption which we call the α -Bruno-Rüssmann condition, and which reduces to the classical Bruno-Rüssmann condition in the analytic category. Our proof is direct in the sense that, for analytic Hamiltonians, we avoid the use of complex extensions and, for non-analytic Hamiltonians, we do not use analytic approximation nor smoothing operators. Following Bessi, we also show that if a slightly weaker arithmetic condition is not satisfied, the invariant torus may be destroyed. Crucial to this work are new functional estimates in the Gevrey class.

Mathematics Subject Classification (2010): 70H08 (primary); 70H09, 26E10, 37C75, 37J40 (secondary).