Generalized stochastic Lagrangian paths
for the Navier-Stokes equation

MARC ARNAUDON, ANA BELA CRUZEIRO AND SHIZAN FANG

Abstract. In the note added in proof of the seminal paper [14], Ebin and Marsden introduced the so-called *correct Laplacian* for the Navier-Stokes equation on a compact Riemannian manifold. In the spirit of Brenier's generalized flows for the Euler equation, we introduce a class of semimartingales on a compact Riemannian manifold. We prove that these semimartingales are critical points to the corresponding kinetic energy if and only if its drift term solves weakly the Navier-Stokes equation defined with Ebin-Marsden's Laplacian. We also show that for the case of torus, classical solutions of the Navier-Stokes equation realize the minimum of the kinetic energy in a suitable class.