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Definition, existence, stability and uniqueness of the solution to a semilinear elliptic problem with a strong singularity at $u = 0$

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Abstract. In this paper we consider a semilinear elliptic equation with a strong singularity at $u = 0$, namely

$$\begin{cases} u \geq 0 & \text{in } \Omega \\ -\operatorname{div} A(x)Du = F(x,u) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $F(x,s)$ is a Carathéodory function such that

$$0 \leq F(x,s) \leq \frac{h(x)}{\Gamma(s)} \quad \text{a.e. } x \in \Omega, \ \forall s > 0$$

with h in some $L^r(\Omega)$ and Γ a $C^1([0,+\infty[)$ function such that $\Gamma(0) = 0$ and $\Gamma'(s) > 0$ for every $s > 0$.

We introduce a notion of solution to this problem in the spirit of the solutions defined by transposition. This definition allows us to prove the existence and the stability of this solution, as well as its uniqueness when $F(x,s)$ is nonincreasing in s.

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