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Global classical solutions in a two-dimensional chemotaxis-Navier-Stokes system with subcritical sensitivity

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Abstract. The chemotaxis-Navier-Stokes system with signal production, as given by

$$\begin{cases} n_t + u \cdot \nabla n = \Delta n - \nabla \cdot \big(nS(x, n, c) \cdot \nabla c \big) \\ c_t + u \cdot \nabla c = \Delta c - c + n \\ u_t + (u \cdot \nabla) u = \Delta u - \nabla P + n \nabla \phi \\ \nabla \cdot u = 0, \end{cases}$$

is considered in bounded planar convex domains Ω with smooth boundary, where $\phi \in W^{2,\infty}(\Omega)$ and $S \in C^2(\bar{\Omega} \times [0, \infty)^2; \mathbb{R}^{2\times 2})$.

The main results assert that parallel to the case of the corresponding Keller-Segel system obtained on neglecting u, any arbitrarily small algebraic saturation effect in the chemotactic sensitivity at large densities is sufficient to rule out any blow-up phenomenon. Indeed, under the assumption that there exist $S_0 \geq 0$ and $\alpha > 0$ such that

$$|S(x, n, c)| \leq S_0 \cdot (1 + n)^{-\alpha} \qquad \text{for all } x \in \bar{\Omega}, n \geq 0 \text{ and } c \geq 0,$$

it is shown that for all suitably regular initial data an associated initial-boundary value problem possesses a globally defined bounded classical solution.

The analysis is based on the consecutive identification of three energy-like functionals, the first among which involves a certain sublinear L^p seminorm of n.

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