

Homotopy classification of ribbon tubes and welded string links

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Abstract. Ribbon 2-knotted objects are locally flat embeddings of surfaces in 4-space which bound immersed 3-manifolds with only ribbon singularities. They appear as topological realizations of the theory of welded knotted objects, which is a natural quotient of virtual knot theory. In this paper we consider ribbon tubes and ribbon torus-links, which are natural analogues of string links and links, respectively. We show how ribbon tubes naturally act on the reduced free group, and how this action classifies ribbon tubes up to link-homotopy, that is when allowing each component to cross itself. At the combinatorial level, this provides a classification of welded string links up to self-virtualization. This generalizes a result of Habegger and Lin on usual string links, and the above-mentioned action on the reduced free group can be refined to a general “virtual extension” of Milnor invariants. As an application, we obtain a classification of ribbon torus-links up to link-homotopy.

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