Frequency of Sobolev dimension distortion of horizontal subgroups in Heisenberg groups

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In memory of Frederick W. Gehring (1925–2012)

Abstract. We study the behavior of Sobolev mappings defined on the sub-Riemannian Heisenberg groups with respect to foliations by left cosets of a horizontal homogeneous subgroup. Our main result provides a quantitative estimate, in terms of Hausdorff dimension, of the size of the set of cosets whose dimension is raised under such mappings. Our approach unifies ideas of Gehring and Mostow about the absolute continuity of quasiconformal mappings with Mattila's projection and slicing machinery.

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