Stable foliations and semi-flow Morse homology

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Abstract. In case of the heat (semi-)flow on the free loop space ΛM of a closed Riemannian manifold M we construct a natural isomorphism between Morse homology and singular homology of ΛM. The construction is not limited to only those semi-flows which are accompanied by a genuine partner flow. (The $W^{1,2}$ partner flow is not used at all in the construction).

There are two main results. Firstly, a method to construct a cellular filtration for the domain of a gradient semi-flow, no background flow needed. Secondly, foliations of Conley pairs. These are of independent interest; see Subsection 1.4 where consequences and perspectives are discussed.

Concerning the natural isomorphism we build a Morse filtration for ΛM using Conley pairs and their pre-images under the time-T-map of the heat flow. The construction is new also in finite dimensions. Due to infinite dimension a subtle step is to contract each Conley pair onto its part in the unstable manifold. To achieve this we construct stable invariant foliations of Conley pairs. It was this step that led to the discovery of a backward λ-lemma [31] for the (forward) heat flow.

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