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Generic finiteness of minimal surfaces with bounded Morse index

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Abstract. Given a compact 3-manifold N without boundary, we prove that for a bumpy metric of positive scalar curvature the space of minimal surfaces having a uniform upper bound on the Morse index is always finite unless the manifold itself contains an embedded minimal $\mathbb{RP}^2$. In particular, we derive a generic finiteness result whenever N does not contain a copy of $\mathbb{RP}^3$ in its prime decomposition. We discuss the obstructions to any further generalization of such a result. When the metric g is required to be (scalar positive and) strongly bumpy (meaning that all closed, immersed minimal surfaces do not have Jacobi fields, a notion recently proved to be generic by B. White) the same conclusion holds true for any closed 3-manifold.

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