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Twisted cohomology of arrangements of lines and Milnor fibers

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Abstract. Let $\mathcal{A}$ be an arrangement of affine lines in $\mathbb{C}^2$, with complement $\mathcal{M}(\mathcal{A})$. The (co)homology of $\mathcal{M}(\mathcal{A})$ with twisted coefficients is strictly related to the cohomology of the Milnor fibre associated to the conified arrangement, endowed with the geometric monodromy. Although several partial results are known, even the first Betti number of the Milnor fiber is not understood. We give here a vanishing conjecture for the first homology, which is of a different nature with respect to the known results. Let Γ be the graph of *double points* of $\mathcal{A}$: we conjecture that if Γ is connected then the geometric monodromy acts trivially on the first homology of the Milnor fiber (so that the first Betti number is combinatorially determined in this case). This conjecture depends only on the combinatorics of $\mathcal{A}$. We prove it in some cases with stronger hypothesis.

In the final parts, we introduce a new description in terms of the group given by the quotient of the commutator subgroup of $\pi_1(\mathcal{M}(\mathcal{A}))$ by the commutator of its *length-zero subgroup*. We use that to deduce some new interesting cases of a-monodromicity, including a proof of the conjecture under some extra conditions.

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