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Monotonicity in half-spaces of positive solutions to $-\Delta_p u = f(u)$ in the case $p > 2$

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Abstract. We consider weak distributional solutions to the equation $-\Delta_p u = f(u)$ in half-spaces under zero Dirichlet boundary condition. We assume that the nonlinearity is positive and superlinear at zero. For $p > 2$ (the case $1 < p \leq 2$ is already known) we prove that any positive solution is strictly monotone increasing in the direction orthogonal to the boundary of the half-space. As a consequence we deduce some Liouville-type theorems for the Lane-Emden-type equation. Furthermore any nonnegative solution turns out to be $C^{2,\alpha}$ smooth.

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