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Propagation of strong singularities in semilinear parabolic equations with degenerate absorption

MOSHE MARCUS AND ANDREY E. SHISHKOV

Abstract. We study equations of the form $(*)$ $u_t - \Delta u + h(x)|u|^{q-1}u = 0$ in a half space $\mathbb{R}_+^{N+1}$. Here $q > 1$ and h is a continuous function in $\mathbb{R}^N$, vanishing at the origin and positive elsewhere. Let $\bar{h}(s) = e^{-\omega(s)/s^2}$ and assume that $\omega(s)/s^2$ is monotone on $(0, 1)$ and tends to infinity as $s \to 0$. We show that, if ω satisfies the Dini condition and $h(x) \geq \bar{h}(|x|)$ then there exists a maximal solution of $(*)$. This solution tends to infinity as $t \to 0$. On the contrary, if the Dini condition in the half space fails and $h(x) \leq \bar{h}(x)$, we construct a sequence of solutions whose initial data shrinks to the Dirac measure with infinite mass at the origin, but the limit of the sequence blows up everywhere on the positive time axis.

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