

On the Dirichlet problem for fully nonlinear elliptic Hessian systems

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Abstract. We consider the problem of existence and uniqueness of strong solutions $u : \Omega \subset \mathbb{R}^n \rightarrow \mathbb{R}^N$ in $(H^2 \cap H_0^1)(\Omega)^N$ to the problem

$$\begin{cases} F(\cdot, D^2u) = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \quad (1)$$

when $f \in L^2(\Omega)^N$, F is a Carathéodory map and Ω is convex. (1) has been considered by several authors, firstly by Campanato and under Campanato's ellipticity condition. By employing a new weaker notion of ellipticity introduced in recent work of the author [25] for the respective global problem on \mathbb{R}^n , we prove well-posedness of (1). Our result extends existing ones under weaker hypotheses than those known previously. An essential part of our analysis is an extension of the classical Miranda-Talenti inequality to the vector case of second order linear Hessian systems with rank-one convex coefficients.

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