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Extensors and the Hilbert scheme

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Abstract. The Hilbert scheme $\mathbf{Hilb}^n_{p(t)}$ parametrizes closed subschemes and families of closed subschemes in the projective space $\mathbb{P}^n$ with a fixed Hilbert polynomial $p(t)$. It can be realized as a closed subscheme of a Grassmannian or a product of Grassmannians. In this paper we consider schemes over a field k of characteristic zero and we present a new proof of the existence of the Hilbert scheme as a subscheme of the Grassmannian $\mathbf{Gr}^{N(r)}_{p(r)}$, where $N(r) = h^0(\mathcal{O}_{\mathbb{P}^n}(r))$. Moreover, we exhibit explicit equations defining it in the Plücker coordinates of the Plücker embedding of $\mathbf{Gr}^{N(r)}_{p(r)}$.

Our proof of existence does not need some of the classical tools used in previous proofs, as flattening stratifications and Gotzmann's Persistence Theorem.

The degree of our equations is $\deg p(t) + 2$, lower than the degree of the equations given by Iarrobino and Kleiman in 1999 and also lower (except for the case of hypersurfaces) than the degree of those proved by Haiman and Sturmfels in 2004 after Bayer's conjecture in 1982.

The novelty of our approach mainly relies on the deeper attention to the intrinsic symmetries of the Hilbert scheme and on some results about Grassmannian based on the notion of extensors.

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