

An over-determined boundary value problem arising from neutrally coated inclusions in three dimensions

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Dedicated to the memory of Professor Kenjiro Okubo

Abstract. We consider the neutral inclusion problem in three dimensions: prove that if a coated inclusion consisting of a core and a shell is neutral to all uniform fields, then the core and the whole inclusion must be concentric balls, if the matrix is isotropic, or confocal ellipsoids if the matrix is anisotropic. We first derive an over-determined boundary value problem in the shell of the neutral inclusion, and then prove in the isotropic case that if the over-determined problem admits a solution, then the core and the whole inclusion must be concentric balls. As a consequence it is proved that the structure is neutral to all uniform fields if and only if it consists of concentric balls provided that the coefficient of the core is larger than that of the shell.

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