

Homological finiteness of Abelian covers

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Abstract. We present a method for deciding when a regular Abelian cover of a finite CW-complex has finite Betti numbers. To start with, we describe a natural parameter space for all regular covers of a finite CW-complex X , with group of deck transformations a fixed Abelian group A , which in the case of free Abelian covers of rank r coincides with the Grassmanian of r -planes in $H^1(X, \mathbb{Q})$. Inside this parameter space, there is a subset $\Omega_A^i(X)$ consisting of all the covers with finite Betti numbers up to degree i .

Building up on work of Dwyer and Fried, we show how to compute these sets in terms of the jump loci for homology with coefficients in rank-1 local systems on X . For certain spaces, such as smooth, quasi-projective varieties, the generalized Dwyer–Fried invariants that we introduce here can be computed in terms of intersections of algebraic subtori in the character group. For many spaces of interest, the homological finiteness of Abelian covers can be tested through the corresponding free Abelian covers. Yet in general, Abelian covers exhibit different homological finiteness properties than their free Abelian counterparts.

Mathematics Subject Classification (2010): 14F35 (primary); 55N25, 20J05, 57M07 (secondary).