On Hilbert's 17th problem and Pfister's multiplicative formulae for the ring of real analytic functions

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Abstract. In this work, we present "infinite" multiplicative formulae for countable collections of sums of squares (of meromorphic functions on $\mathbb{R}^n$). Our formulae generalize the classical Pfister's ones concerning the representation as a sum of 2^r squares of the product of two elements of a field K which are sums of 2^r squares. As a main application, we reduce the representation of a positive semidefinite analytic function on $\mathbb{R}^n$ as a sum of squares to the representation as sums of squares of its *special factors*. Recall that roughly speaking a special factor is an analytic function on $\mathbb{R}^n$ which has just one complex irreducible factor and whose zeroset has dimension between 1 and $n-2$.