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A structural theorem for codimension-one foliations on $\mathbb{P}^n$, $n \geq 3$, with an application to degree-three foliations

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Abstract. Let $\mathcal{F}$ be a codimension-one foliation on $\mathbb{P}^n$: for each point $p \in \mathbb{P}^n$ we define $\mathcal{J}(\mathcal{F}, p)$ as the order of the first non-zero jet $j_p^k(\omega)$ of a holomorphic 1-form ω defining $\mathcal{F}$ at p. The singular set of $\mathcal{F}$ is $\text{sing}(\mathcal{F}) = \{p \in \mathbb{P}^n \mid \mathcal{J}(\mathcal{F}, p) \geq 1\}$. We prove (main Theorem 1.2) that a foliation $\mathcal{F}$ satisfying $\mathcal{J}(\mathcal{F}, p) \leq 1$ for all $p \in \mathbb{P}^n$ has a non-constant rational first integral. Using this fact we are able to prove that any foliation of degree-three on $\mathbb{P}^n$, with $n \geq 3$, is either the pull-back of a foliation on $\mathbb{P}^2$, or has a transverse affine structure with poles. This extends previous results for foliations of degree at most two.

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