

Torsion points on elliptic curves in Weierstrass form

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Abstract. We prove that there are only finitely many complex numbers a and b with $4a^3 + 27b^2 \neq 0$ such that the three points $(1, *)$, $(2, *)$, and $(3, *)$ are simultaneously torsion points on the elliptic curve defined in Weierstrass form by $y^2 = x^3 + ax + b$. This gives an affirmative answer to a question raised by Masser and Zannier. We thus confirm a special case in two dimensions of the relative Manin-Mumford Conjecture formulated by Pink and Masser-Zannier.

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