Compact sets with vanishing cohomology in Stein spaces and domains of holomorphy

VIOREL VÂJÂITU

Abstract. Let X be a Stein space. We study compact subsets K of X that are structurally acyclic, *i.e.*, $H^i(K, \mathcal{O}_X) = 0$, for all $i \geq 1$. We show i) that such compact sets are *natural* in the sense that the canonical map from K into $\widetilde{K}$, the spectrum of the complex algebra $\Gamma(K, \mathcal{O}_X)$, is bijective, and ii) that the set of interior points of K is a domain of holomorphy in X. Motivated by this we give an extensive account of examples of domains of holomorphy in non-normal Stein spaces and prove several properties, like hereditarity via the normalization map. Finally, a straightforward criterion of non-acyclicity is given in terms of general Hartogs figures.

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