Saddle-shaped solutions of bistable elliptic equations involving the half-Laplacian

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Abstract. We establish existence and qualitative properties of saddle-shaped solutions of the elliptic fractional equation $(-\Delta)^{1/2}u = f(u)$ in the whole space $\mathbb{R}^{2m}$, where f is of bistable type. These solutions are odd with respect to the Simons cone and even with respect to each coordinate.

More precisely, we prove the existence of a saddle-shaped solution in every even dimension $2m$, as well as its monotonicity properties, asymptotic behaviour, and instability in dimensions $2m = 4$ and $2m = 6$.

These results are relevant in connection with the analog for fractional equations of a conjecture of De Giorgi on the 1-D symmetry of certain solutions. Saddle-shaped solutions are the simplest candidates, besides 1-D solutions, to be global minimizers in high dimensions, a property not yet established.