On plane rational curves and the splitting of the tangent bundle

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Abstract. Given an immersion $\varphi : \mathbf{P}^1 \to \mathbf{P}^2$, we give new approaches to determining the splitting of the pullback of the cotangent bundle. We also give new bounds on the splitting type for immersions which factor as $\varphi : \mathbf{P}^1 \cong D \subset X \to \mathbf{P}^2$, where $X \to \mathbf{P}^2$ is obtained by blowing up r distinct points $p_i \in \mathbf{P}^2$. As applications in the case that the points p_i are generic, we give a complete determination of the splitting types for such immersions when $r \leq 7$. The case that $D^2 = -1$ is of particular interest. For $r \leq 8$ generic points, it is known that there are only finitely many inequivalent φ with $D^2 = -1$, and all of them have balanced splitting. However, for $r = 9$ generic points we show that there are infinitely many inequivalent φ with $D^2 = -1$ having unbalanced splitting (only two such examples were known previously). We show that these new examples are related to a semi-adjoint formula which we conjecture accounts for all occurrences of unbalanced splitting when $D^2 = -1$ in the case of $r = 9$ generic points p_i. In the last section we apply such results to the study of the resolution of fat point schemes.

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