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Non-divergence form parabolic equations associated with non-commuting vector fields: boundary behavior of nonnegative solutions

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Abstract. In a cylinder $\Omega_T = \Omega \times (0, T) \subset \mathbb{R}^{n+1}_+$ we study the boundary behavior of nonnegative solutions of second order parabolic equations of the form

$$Hu = \sum_{i,j=1}^{m} a_{ij}(x,t) X_i X_j u - \partial_t u = 0, \ (x,t) \in \mathbb{R}^{n+1}_+,$$

where $X = \{X_1, \dots, X_m\}$ is a system of C^∞ vector fields in $\mathbb{R}^n$ satisfying Hörmander's rank condition (1.2), and Ω is a non-tangentially accessible domain with respect to the Carnot-Carathéodory distance d induced by X. Concerning the matrix-valued function $A = \{a_{ij}\}$, we assume that it is real, symmetric and uniformly positive definite. Furthermore, we suppose that its entries a_{ij} are Hölder continuous with respect to the parabolic distance associated with d. Our main results are: 1) a backward Harnack inequality for nonnegative solutions vanishing on the lateral boundary (Theorem 1.1); 2) the Hölder continuity up to the boundary of the quotient of two nonnegative solutions which vanish continuously on a portion of the lateral boundary (Theorem 1.2); 3) the doubling property for the parabolic measure associated with the operator H (Theorem 1.3). These results generalize to the subelliptic setting of the present paper, those in Lipschitz cylinders by Fabes, Safonov and Yuan in [20, 39]. With one proviso: in those papers the authors assume that the coefficients a_{ij} be only bounded and measurable, whereas we assume Hölder continuity with respect to the intrinsic parabolic distance.

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