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Ordinary holomorphic webs of codimension one

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Abstract. To any d-web of codimension one on a holomorphic n-dimensional manifold M $(d > n)$, we associate an analytic subset S of M. We call **ordinary** the webs for which S has a dimension at most $n - 1$ or is empty. This condition is generically satisfied, at least at the level of germs.

We prove that the rank of an ordinary d-web has an upper-bound $\pi'(n, d)$ which, for $n \geq 3$, is strictly smaller than the bound $\pi(n, d)$ proved by Chern, $\pi(n, d)$ denoting the Castelnuovo's number. This bound is optimal.

Setting $c(n,h) = \binom{n-1+h}{h}$, let k_0 be the integer such that $c(n,k_0) \leq d < c(n, k_0+1)$. The number $\pi'(n, d)$ is then equal

- to 0 for $d < c(n, 2)$,
- and to $\sum_{h=1}^{k_0} \big(d - c(n, h)\big)$ for $d \geq c(n, 2)$.

Moreover, if d is precisely equal to $c(n, k_0)$, we define off S a holomorphic connection on a holomorphic bundle $\mathcal{E}$ of rank $\pi'(n, d)$, such that the set of Abelian relations off S is isomorphic to the set of holomorphic sections of $\mathcal{E}$ with vanishing covariant derivative: the curvature of this connection, which generalizes the Blaschke curvature, is then an obstruction for the rank of the web to reach the value $\pi'(n, d)$.

When $n=2$, S is always empty so that any web is ordinary, $\pi'(2,d) = \pi(2,d)$, and any d may be written $c(2, k_0)$: we recover the results given in [9].

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