

Closed surfaces with bounds on their Willmore energy

ERNST KUWERT AND REINER SCHÄTZLE

Abstract. The Willmore energy of a closed surface in \mathbb{R}^n is the integral of its squared mean curvature, and is invariant under Möbius transformations of \mathbb{R}^n . We show that any torus in \mathbb{R}^3 with energy at most $8\pi - \delta$ has a representative under the Möbius action for which the induced metric and a conformal metric of constant (zero) curvature are uniformly equivalent, with constants depending only on $\delta > 0$. An analogous estimate is also obtained for closed, orientable surfaces of fixed genus $p \geq 1$ in \mathbb{R}^3 or \mathbb{R}^4 , assuming suitable energy bounds which are sharp for $n = 3$. Moreover, the conformal type is controlled in terms of the energy bounds.

Mathematics Subject Classification (2010): 53A05 (primary); 53A30, 53C21, 49Q15 (secondary).