Nonuniformly hyperbolic cocycles: admissibility and robustness

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Abstract. We give a relatively short proof of the robustness of nonuniformly hyperbolic cocycles in a Banach space under sufficiently small perturbations. In strong contrast to former proofs, we do not need to construct projections leading to the stable and unstable subspaces. Instead, these are obtained fairly explicitly depending only on the boundedness respectively of forward and backward orbits. A difficulty is that we need to construct from the beginning appropriate sequences of Lyapunov norms, with respect to which one can measure the boundedness of the orbits. These norms need not only to be guessed a priori but also all the computations would change if these were not appropriate, both for the original and for the perturbed cocycles. The proof of the robustness is based on the relation between the notions of nonuniform exponential dichotomy and of admissibility, together with nontrivial norm bounds for the expansion and contraction and for the norms of the projections. This relation allows us to construct an invertible operator from the set of bounded perturbations to the set of bounded solutions, and thus to conclude that under sufficiently small perturbations a similar operator exists for the perturbed cocycle.

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