

**Forward, backward and elliptic Harnack inequalities  
for non-negative solutions  
to certain singular parabolic partial differential equations**

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**Abstract.** Forward, backward and elliptic Harnack inequalities for non-negative solutions of a class of singular, quasi-linear, parabolic equations, are established. These classes of singular equations include the  $p$ -Laplacean equation and equations of the porous medium type. Key novel points include form of a Harnack estimate backward in time, that has never been observed before, and measure theoretical proofs, as opposed to comparison principles. These Harnack estimates are established in the super-critical range (1.5) below. Such a range is optimal for a Harnack estimate to hold.

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