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H^1 and *BMO* for certain locally doubling metric measure spaces

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Abstract. Suppose that (M,ρ,μ) is a metric measure space, which possesses two "geometric" properties, called "isoperimetric" property and approximate midpoint property, and that the measure μ is locally doubling. The isoperimetric property implies that the volume of balls grows at least exponentially with the radius. Hence the measure μ is not globally doubling. In this paper we define an atomic Hardy space $H^1(\mu)$, where atoms are supported only on "small balls", and a corresponding space $BMO(\mu)$ of functions of "bounded mean oscillation", where the control is only on the oscillation over small balls. We prove that $BMO(\mu)$ is the dual of $H^1(\mu)$ and that an inequality of John–Nirenberg type on small balls holds for functions in $BMO(\mu)$. Furthermore, we show that the $L^p(\mu)$ spaces are intermediate spaces between $H^1(\mu)$ and $BMO(\mu)$, and we develop a theory of singular integral operators acting on function spaces on M. Finally, we show that our theory is strong enough to give $H^1(\mu)$-$L^1(\mu)$ and $L^\infty(\mu)$-$BMO(\mu)$ estimates for various interesting operators on Riemannian manifolds and symmetric spaces which are unbounded on $L^1(\mu)$ and on $L^\infty(\mu)$.

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