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On the second boundary value problem for Monge-Ampère type equations and optimal transportation

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Abstract. This paper is concerned with the existence of globally smooth solutions for the second boundary value problem for certain Monge-Ampère type equations and the application to regularity of potentials in optimal transportation. In particular we address the fundamental issue of determining conditions on costs and domains to ensure that optimal mappings are smooth diffeomorphisms. The cost functions satisfy a weak form of the condition (A3), which was introduced in a recent paper with Xi-nan Ma, in conjunction with interior regularity. Our condition is optimal and includes the quadratic cost function case of Caffarelli and Urbas as well as the various examples in our previous work. The approach is through the derivation of global estimates for second derivatives of solutions.

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