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## A family of adapted complexifications for $SL_2(\mathbb{R})$

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**Abstract.** Let *G* be a non-compact, real semisimple Lie group. We consider maximal complexifications of *G* which are adapted to a distinguished oneparameter family of naturally reductive, left-invariant metrics. In the case of  $G = SL_2(\mathbb{R})$  their realization as equivariant Riemann domains over  $G^{\mathbb{C}} = SL_2(\mathbb{C})$  is carried out and their complex-geometric properties are investigated. One obtains new examples of non-univalent, non-Stein, maximal adapted complexifications.

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