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Sobolev regularity via the convergence rate of convolutions and Jensen's inequality

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Abstract. We derive a new criterion for a real-valued function u to be in the Sobolev space $W^{1,2}(\mathbb{R}^n)$. This criterion consists of comparing the value of a functional $\int f(u)$ with the values of the same functional applied to convolutions of u with a Dirac sequence. The difference of these values converges to zero as the convolutions approach u, and we prove that the rate of convergence to zero is connected to regularity: $u \in W^{1,2}$ if and only if the convergence is sufficiently fast. We finally apply our criterium to a minimization problem with constraints, where regularity of minimizers cannot be deduced from the Euler-Lagrange equation.

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