

## Extensions of the Cugiani-Mahler theorem

YANN BUGEAUD

**Abstract.** In 1955, Roth established that if  $\xi$  is an irrational number such that there are a positive real number  $\varepsilon$  and infinitely many rational numbers  $p/q$  with  $q \geq 1$  and  $|\xi - p/q| < q^{-2-\varepsilon}$ , then  $\xi$  is transcendental. A few years later, Cugiani obtained the same conclusion with  $\varepsilon$  replaced by a function  $q \mapsto \varepsilon(q)$  that decreases very slowly to zero, provided that the sequence of rational solutions to  $|\xi - p/q| < q^{-2-\varepsilon(q)}$  is sufficiently dense, in a suitable sense. We give an alternative, and much simpler, proof of Cugiani's Theorem and extend it to simultaneous approximation.

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