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Equations in the Hadamard ring of rational functions

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Abstract. Let K be a number field. It is well known that the set of recurrence-sequences with entries in K is closed under component-wise operations, and so it can be equipped with a ring structure. We try to understand the structure of this ring, in particular to understand which algebraic equations have a solution in the ring. For the case of cyclic equations a conjecture due to Pisot states the following: assume $\{a_n\}$ is a recurrence sequence and suppose that all the a_n have a d^{th} root in the field K; then (after possibly passing to a finite extension of K) one can choose a sequence of such d^{th} roots that satisfies a recurrence itself. This was proved true in a preceding paper of the second author. In this article we generalize this result to more general monic equations; the former case can be recovered for $g(X, Y) = X^d - Y = 0$. Combining this with the *Hadamard quotient theorem* by Pourchet and Van der Poorten, we are able to get rid of the monic restriction, and have a theorem that generalizes both results.

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