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The equation $-\Delta u - \lambda \dfrac{u}{|x|^2} = |\nabla u|^p + cf(x)$: The optimal power

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Abstract. We will consider the following problem

$$-\Delta u - \lambda \frac{u}{|x|^2} = |\nabla u|^p + c\, f, \quad u > 0 \text{ in } \Omega,$$

where $\Omega \subset \mathbb{R}^N$ is a domain such that $0 \in \Omega$, $N \geq 3$, $c > 0$ and $\lambda > 0$. The main objective of this note is to study the precise threshold $p_+ = p_+(\lambda)$ for which there is no *very weak supersolution* if $p \geq p_+(\lambda)$. The optimality of $p_+(\lambda)$ is also proved by showing the solvability of the Dirichlet problem when $1 \leq p < p_+(\lambda)$, for $c > 0$ small enough and $f \geq 0$ under some hypotheses that we will prescribe.

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