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Multivariate moment problems: Geometry and indeterminateness

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Abstract. The most accurate determinateness criteria for the multivariate moment problem require the density of polynomials in a weighted Lebesgue space of a generic representing measure. We propose a relaxation of such a criterion to the approximation of a single function, and based on this condition we analyze the impact of the geometry of the support on the uniqueness of the representing measure. In particular we show that a multivariate moment sequence is determinate if its support has dimension one and is virtually compact; a generalization to higher dimensions is also given. Among the one-dimensional sets which are not virtually compact, we show that at least a large subclass supports indeterminate moment sequences. Moreover, we prove that the determinateness of a moment sequence is implied by the same condition (in general easier to verify) of the push-forward sequence via finite morphisms.

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