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On the space of morphisms into generic real algebraic varieties

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Abstract. We introduce a notion of generic real algebraic variety and we study the space of morphisms into these varieties. Let Z be a real algebraic variety. We say that Z is generic if there exist a finite family $\{D_i\}_{i=1}^n$ of irreducible real algebraic curves with genus ≥ 2 and a biregular embedding of Z into the product variety $\prod_{i=1}^n D_i$. A bijective map $\varphi : \widetilde{Z} \to Z$ from a real algebraic variety $\widetilde{Z}$ to Z is called weak change of the algebraic structure of Z if it is regular and its inverse is a Nash map. Generic real algebraic varieties are "generic" in the sense specified by the following result: For each real algebraic variety Z and for integer k, there exists an algebraic family $\{\varphi_t : \widetilde{Z}_t \to Z\}_{t \in \mathbb{R}^k}$ of weak changes of the algebraic structure of Z such that $\widetilde{Z}_0 = Z$, φ_0 is the identity map on Z and, for each $t \in \mathbb{R}^k \setminus \{0\}$, $\widetilde{Z}_t$ is generic. Let X and Y be nonsingular irreducible real algebraic varieties. Regard the set $\mathcal{R}(X, Y)$ of regular maps from X to Y as a subspace of the corresponding set $\mathcal{N}(X, Y)$ of Nash maps, equipped with the C^∞ compact-open topology. We prove that, if Y is generic, then $\mathcal{R}(X, Y)$ is closed and nowhere dense in $\mathcal{N}(X, Y)$, and has a semi-algebraic structure. Moreover, the set of dominating regular maps from X to Y is finite. A version of the preceding results in which X and Y can be singular is given also.

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