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On the arithmetic properties of complex values of Hecke-Mahler series I. The rank one case

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Abstract. Here we characterise, in a complete and explicit way, the relations of algebraic dependence over \mathbb{Q} of complex values of Hecke-Mahler series taken at algebraic points $\underline{u}_1, \dots, \underline{u}_m$ of the multiplicative group $\mathbb{G}_m^2(\mathbb{C})$, under a technical hypothesis that a certain sub-module of $\mathbb{G}_m^2(\mathbb{C})$ generated by the \underline{u}_i 's has rank one (rank one hypothesis). This is the first part of a work, announced in [Pel1], whose main objective is completely to solve a general problem on the algebraic independence of values of these series.

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