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A criterion for virtual global generation

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Abstract. Let X be a smooth projective curve defined over an algebraically closed field k, and let F_X denote the absolute Frobenius morphism of X when the characteristic of k is positive. A vector bundle over X is called virtually globally generated if its pull back, by some finite morphism to X from some smooth projective curve, is generated by its global sections. We prove the following. If the characteristic of k is positive, a vector bundle E over X is virtually globally generated if and only if $(F_X^m)^*E \cong E_a \oplus E_f$ for some m, where E_a is some ample vector bundle and E_f is some finite vector bundle over X (either of E_a and E_f are allowed to be zero). If the characteristic of k is zero, a vector bundle E over X is virtually globally generated if and only if E is a direct sum of an ample vector bundle and a finite vector bundle over X (either of them are allowed to be zero).

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