

Linear independence of linear forms in polylogarithms

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Abstract. For $x \in \mathbb{C}$, $|x| < 1$, $s \in \mathbb{N}$, let $\text{Li}_s(x)$ be the s -th polylogarithm of x . We prove that for any non-zero algebraic number α such that $|\alpha| < 1$, the $\mathbb{Q}(\alpha)$ -vector space spanned by $1, \text{Li}_1(\alpha), \text{Li}_2(\alpha), \dots$ has infinite dimension. This result extends a previous one by Rivoal for rational α . The main tool is a method introduced by Fischler and Rivoal, which shows the coefficients of the polylogarithms in the relevant series to be the unique solution of a suitable Padé approximation problem.

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