Hardy-type inequalities related to
degenerate elliptic differential operators

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Abstract. We prove some Hardy-type inequalities related to quasilinear second-order degenerate elliptic differential operators $L_p u := -\nabla_L^*(|\nabla_L u|^{p-2} \nabla_L u)$. If ϕ is a positive weight such that $-L_p\phi \geq 0$, then the Hardy-type inequality

$$c \int_\Omega \frac{|u|^p}{\phi^p} |\nabla_L \phi|^p \, d\xi \leq \int_\Omega |\nabla_L u|^p \, d\xi \qquad \left(u \in \mathscr{C}_0^1(\Omega) \right)$$

holds. We find an explicit value of the constant involved, which, in most cases, results optimal. As particular case we derive Hardy inequalities for subelliptic operators on Carnot Groups.