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Hardy-type inequalities related to degenerate elliptic differential operators

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Abstract. We prove some Hardy-type inequalities related to quasilinear secondorder degenerate elliptic differential operators $L_p u := -\nabla_L^* (|\nabla_L u|^{p-2} \nabla_L u)$. If ϕ is a positive weight such that $-L_p \phi \ge 0$, then the Hardy-type inequality

$$c\int_{\Omega} \frac{|u|^p}{\phi^p} |\nabla_{\!L} \phi|^p \ d\xi \leq \int_{\Omega} |\nabla_{\!L} u|^p \ d\xi \qquad \left(u \in \mathscr{C}^1_0(\Omega)\right)$$

holds. We find an explicit value of the constant involved, which, in most cases, results optimal. As particular case we derive Hardy inequalities for subelliptic operators on Carnot Groups.

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