Ann. Scuola Norm. Sup. Pisa Cl. Sci. (5)
Vol. IV (2005), 187-217

# Heat flows for extremal Kähler metrics

SANTIAGO R. SIMANCA

**Abstract.** Let $(M, J, \Omega)$ be a closed polarized complex manifold of Kähler type. Let $G$ be the maximal compact subgroup of the automorphism group of $(M, J)$. On the space of Kähler metrics that are invariant under $G$ and represent the cohomology class $\Omega$, we define a flow equation whose critical points are the extremal metrics, *i.e.* those that minimize the square of the $L^2$-norm of the scalar curvature. We prove that the dynamical system in this space of metrics defined by the said flow does not have periodic orbits, and that its only fixed points are the extremal metrics. We prove local time-existence of the flow, and conclude that if the lifespan of the solution is finite, then the supremum of the norm of its curvature tensor must blow up as time approaches it. While doing this, we also prove that extremal solitons can only exist in the non-compact case, and that the range of the holomorphy potential of the scalar curvature is an interval independent of the metric chosen to represent $\Omega$. We end up with some conjectures concerning the plausible existence and convergence of global solutions under suitable geometric conditions.

**Mathematics Subject Classification (2000):** 53C55 (primary); 35K55, 58E11, 58J35 (secondary).