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Free Boundary Problems and Transonic Shocks for the Euler Equations in Unbounded Domains

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Abstract. We establish the existence and stability of multidimensional transonic shocks (hyperbolic-elliptic shocks), which are not nearly orthogonal to the flow direction, for the Euler equations for steady compressible potential fluids in unbounded domains in $\mathbb{R}^n$, $n \geq 3$. The Euler equations can be written as a second order nonlinear equation of mixed hyperbolic-elliptic type for the velocity potential. The transonic shock problem can be formulated into the following free boundary problem: The free boundary is the location of the multidimensional transonic shock which divides two regions of $C^{2,\alpha}$ flow, and the equation is hyperbolic in the upstream region where the $C^{2,\alpha}$ perturbed flow is supersonic. In this paper, we develop a new approach to deal with such free boundary problems and establish the existence and stability of multidimensional transonic shocks near planes. We first reformulate the free boundary problem into a fixed conormal boundary value problem for a nonlinear elliptic equation of second order in unbounded domains and then develop techniques to solve this elliptic problem. Our results indicate that there exists a solution of the free boundary problem such that the equation is always elliptic in the unbounded downstream region, the uniform velocity state at infinity in the downstream direction is equal to the unperturbed downstream velocity state, and the free boundary is $C^{2,\alpha}$, provided that the hyperbolic phase is close in $C^{2,\alpha}$ to a uniform flow. We further prove that the free boundary is stable under the $C^{2,\alpha}$ steady perturbation of the hyperbolic phase. Moreover, we extend our existence results to the case that the regularity of the steady perturbation is only $C^{1,1}$, and we introduce another simpler approach to deal with the existence and stability problem when the regularity of the steady perturbation is $C^{3,\alpha}$ or higher. We also establish the existence and stability of multidimensional transonic shocks near spheres in $\mathbb{R}^n$.

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