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A Strongly Degenerate Quasilinear Equation: the Elliptic Case

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Abstract. We prove existence and uniqueness of entropy solutions for the Neumann problem for the quasilinear elliptic equation $u - \text{div } \mathbf{a}(u, Du) = v$, where $v \in L^1$, $\mathbf{a}(z, \xi) = \nabla_{\xi} f(z, \xi)$, and f is a convex function of ξ with linear growth as $\|\xi\| \to \infty$, satisfying other additional assumptions. In particular, this class includes the case where $f(z, \xi) = \varphi(z)\psi(\xi)$, $\varphi > 0$, ψ being a convex function with linear growth as $\|\xi\| \to \infty$. In the second part of this work, using Crandall-Ligget's iteration scheme, this result will permit us to prove existence and uniqueness of entropy solutions for the corresponding parabolic problem with initial data in L^1 .

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