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Boundary Trace of Positive Solutions of Nonlinear Elliptic Inequalities

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Abstract. We develop a new method for proving the existence of a boundary trace, in the class of Borel measures, of nonnegative solutions of $-\Delta u + g(x, u) \geq 0$ in a smooth domain Ω under very general assumptions on g. This new definition which extends the previous notions of boundary trace is based upon a sweeping technique by solutions of Dirichlet problems with measure boundary data. We also prove a boundary pointwise blow-up estimate of any solution of such inequalities in terms of the Poisson kernel. If the nonlinearity is very degenerate near the boundary, for example if $g(x, u) \approx \exp(-\rho_{\partial\Omega}^{-1}(x))u^q$, we exhibit a new full boundary blow-up phenomenon.

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