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Dehn surgery and Seifert surface systems

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Abstract. For a compact connected 3-submanifold with connected boundary in the 3-sphere, we relate the existence of a Seifert surface system with properties of Dehn surgeries along null-homologous links. As a corollary, we obtain a refinement of Fox's re-embedding theorem, and show the existence of a Seifert surface system for any closed surface in the 3-sphere.

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1. Introduction

Seifert surfaces for a knot in the 3-sphere, which were introduced in [4] and [18], have played a central role in knot theory. Embeddings of a handlebody or closed surface into the 3-sphere can be regarded as a natural generalization of knots. In this paper we consider Seifert surface systems for a handlebody embedded in the 3-sphere, and determine when completely disjoint Seifert surface systems exist by means of Dehn surgeries. This gives a new characterization for the existence of such completely disjoint surface systems, supplementing the results in [8, 10] and [7]. See Remark 1.7 below for other equivalent conditions.

Definition 1.1. Let M be a compact connected 3-manifold with connected boundary of genus g. A *spanning surface system* $\{F_i\}$ for M is a set satisfying the following:

(1) $\{F_i\}$ is a set of disjoint orientable surfaces without closed components which are properly embedded in M;
(2) $\{\partial F_i\}$ is a set of g disjoint loops $C_1, \ldots, C_g$ which do not separate ∂M.

A spanning surface system $\{F_i\}$ for M is *completely disjoint* if $\{F_i\}$ is a set of g disjoint orientable surfaces.

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Remark 1.2. By [11, Corollary 1.4], it follows that if M is a handlebody and $\{F_i\}$ is a completely disjoint spanning surface system for M, then there exists a meridian disk system $\{D_i\}$ for M such that $\partial F_i = \partial D_i$ for $i = 1, \ldots, g$.

By a homological argument, we have the following:

Proposition 1.3. *Any compact connected* 3*-submanifold with connected boundary in* S^3 *admits a spanning surface system.*

However, a compact connected 3-submanifold with connected boundary in S^3 does not always admit a completely disjoint spanning surface system; see [10] for a genus 2 example.

Next we consider Seifert surface systems for a closed surface.

Definition 1.4. Let S be a genus $g > 0$ closed surface in S^3, and define $S^3 = V \cup_S V'$. A *Seifert surface system* $(\{F_i\}, \{F'_i\})$ for S is a pair of sets satisfying the following.

(1) $\{F_i\}$ (respectively $\{F'_i\}$) is a spanning surface system for V (respectively V');
(2) $|C_i \cap C'_j| = \delta_{ij}$ for $i, j = 1, \ldots, g$, where $\{\partial F_i\} = \{C_1, \ldots, C_g\}$ and $\{\partial F'_i\} = \{C'_1, \ldots, C'_g\}$.

A Seifert surface system $(\{F_i\}, \{F'_i\})$ for S is *completely disjoint* if $\{F_i\}$ and $\{F'_i\}$ are completely disjoint.

Definition 1.5. Let $L = L_1 \cup \cdots \cup L_n$ be a link in S^3. Following [12], we say that L is a *reflexive link* if the 3-sphere can be obtained by a non-trivial Dehn surgery along L. In particular, if the surgery slope γ_i for L_i is $1/n_i$ for some integer n_i $(i = 1, \ldots, n)$, then we call the Dehn surgery a $1/\mathbb{Z}$*-Dehn surgery*.

Suppose that L is contained in a compact 3-submanifold M in S^3. We say that L is *null-homologous* in M if $[L] = 0$ in $H_1(M; \mathbb{Z})$, and that L is *completely null-homologous* in M if $[L_i] = 0$ in $H_1(M; \mathbb{Z})$ for $i = 1, \ldots, n$.

Theorem 1.6. *Let M be a compact connected* 3*-submanifold with connected boundary in* S^3*. Then the following hold:*

(1) *there exists a null-homologous link L in M, which is reflexive in S^3, such that a handlebody can be obtained from M by a* $1/\mathbb{Z}$*-Dehn surgery along L;*
(2) *M admits a completely disjoint spanning surface system if and only if there exists a completely null-homologous link L in M, which is reflexive in S^3, such that a handlebody can be obtained from M by a* $1/\mathbb{Z}$*-Dehn surgery along L.*

We remark that in Theorem 1.6 (2), we can take a completely null-homologous reflexive link L so that it is disjoint from the completely disjoint spanning surface system.

Theorem 1.6 can be applied to tangle spaces as follows. Lambert showed in [10] that the tangle space of the square tangle admits no completely disjoint spanning surface system. With Theorem 1.6 (2), this result implies that a handlebody cannot be obtained from the tangle space by a $1/\mathbb{Z}$-Dehn surgery along any completely null-homologous link which is reflexive in S^3.

Remark 1.7. Let M be a compact connected 3-submanifold of S^3 with connected boundary of genus g. Let $f : M \to X$ be a map onto a genus g handlebody X. We say that f is a *boundary preserving map* of M onto X if f is continuous and $f|_{\partial M}$ is a homeomorphism onto ∂X. We say that M is *retractable* if M can be retracted onto a wedge of g simple closed curves. If such a wedge can be chosen to be in ∂M, then M is called *boundary retractable*. Set $G = \pi_1(M)$ and define $G_1 = [G, G]$, $G_{n+1} = [G_n, G]$, $G_\omega = \bigcap_n G_n$. Then the following conditions are equivalent:

(1) M admits a completely disjoint spanning surface system;
(2) there exists a boundary preserving map from M onto a handlebody;
(3) M is boundary retractable;
(4) the natural map $\pi_1(\partial M) \to G/G_\omega$ is an epimorphism.

The equivalence between (1) and (2) was shown in [10, Theorem 2]. The equivalence between (2) and (3) was shown in [8, Theorem 3]. The equivalence between (3) and (4) was shown in [7, Theorem 2, 3].

Let M be a compact connected 3-submanifold of S^3. By Proposition 1.3, each component of the exterior of M admits a spanning surface system. If we apply Theorem 1.6 (1) to every component of the exterior of M, then we obtain the following refinement of Fox's re-embedding Theorem.

Corollary 1.8 ([3,14,17]). *Every compact connected* 3*-submanifold M of S^3 can be re-embedded in S^3 so that the exterior of the image of M is a union of handlebodies.*

Remark 1.9. In relation with Remark 1.7, there is an another equivalent condition. Let M be a compact connected 3-submanifold of S^3 with connected boundary of genus 2. By Corollary 1.8, there exists a re-embedding of M so that its exterior is a genus 2 handlebody V. A handcuff graph shaped spine Γ of V is a *boundary spine* if its constituent link L_Γ is a boundary link that admits a pair of disjoint Seifert surfaces whose interiors are contained in $S^3 - \Gamma$. A handlebody V is $(3)_S$*-knotted* if it does not admit any boundary spine. Then it was shown in [1, Theorem 3.10] that M admits a completely disjoint spanning surface system if and only if H is not $(3)_S$-knotted.

Corollary 1.10. *Let S be a closed surface in S^3 which separates S^3 into* 3*-submanifolds M and M' Then the following hold:*

(1) *there exist null-homologous links L in M and L' in M', which are reflexive in S^3, such that handlebodies can be obtained from M and M' by $1/\mathbb{Z}$-Dehn surgeries along L and L';*
(2) *S admits a completely disjoint Seifert surface system if and only if there exist completely null-homologous links L in M and L' in M', which are reflexive in S^3, such that handlebodies can be obtained from M and M' by $1/\mathbb{Z}$-Dehn surgeries along L and L'.*

By Corollary 1.10 (1), we can obtain a Seifert surface system from a meridian-longitude disk system for the handlebodies by tubing along the null-homologous links.

Corollary 1.11. *Any closed surface in S^3 admits a Seifert surface system.*

Let M be a 3-manifold and let $L \subset M$ be a submanifold with or without boundary. When L is 1 or 2-dimensional, we write $E(L) = M - int\, N(L)$, and when L is 3-dimensional, we write $E(L) = M - int\, L$.

2. Proof

Let V be a genus g handlebody in S^3, and $\{D_i\}$ be a meridian disk system for V. Since $V - \bigcup_i int\, N(D_i)$ is a 3-ball, there exists a spine Γ of V such that:

(1) Γ consists of g loops $l_1, \dots, l_g$ and g arcs $\gamma_1, \dots, \gamma_g$ connecting l_i to a point x;
(2) The point x is in the interior of the 3-ball $V - \bigcup_i int\, N(D_i)$, which is homeomorphic to $N(x \cup \gamma_1 \cup \cdots \cup \gamma_g)$;
(3) Each loop l_i is dual to D_i.

We call this spine Γ a *g-handcuff graph shaped spine* for V with respect to $\{D_i\}$.

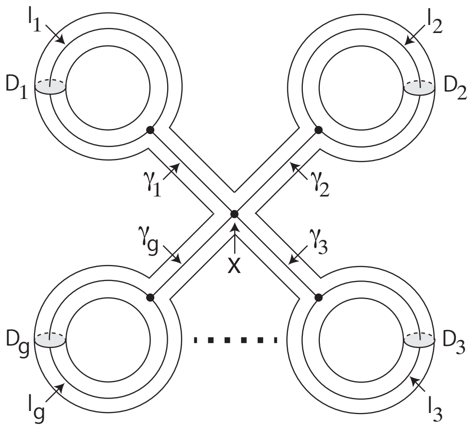


Figure 2.1. A g-handcuff graph shaped spine for V with respect to $\{D_i\}$.

Next, let $\{F_i\}$ be a set of orientable surfaces with boundary and without closed components. We say that $\{F_i\}$ is a *Seifert surface system* for Γ if $(\bigcup_i F_i) \cap \Gamma = \bigcup_i \partial F_i = \bigcup_i l_i$.

Lemma 2.1. *Any g-handcuff graph shaped spine in S^3 admits a Seifert surface system.*

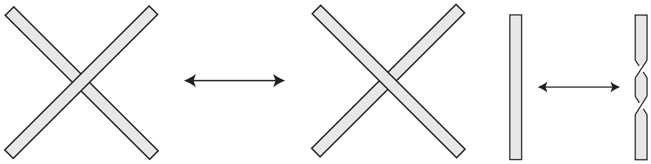
Proof. We take a regular diagram of Γ such that $x \cup \gamma_1 \cup \cdots \cup \gamma_g$ has no crossing. Then we apply the Seifert's algorithm [18] to the loops $l_1 \cup \cdots \cup l_g$ with arbitrary orientations, and obtain Seifert surfaces $\{F_i'\}$ for the loops. □

The following lemma states that from any meridian disk system for a handlebody we can obtain a Seifert surface system for the boundary of the handlebody.

Lemma 2.2. *Let V be a genus g handlebody in S^3 with a meridian disk system $\{D_i\}$. Then there exists a spanning surface system $\{F_i\}$ for $E(V)$ such that $(\{D_i\}, \{F_i\})$ is a Seifert surface system for ∂V.*

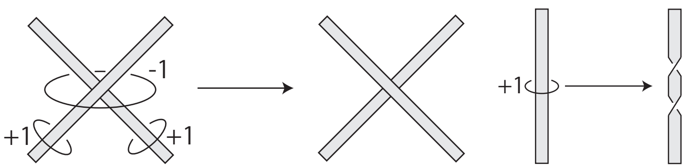
Proof. Let Γ be a g-handcuff graph shaped spine Γ for V with respect to $\{D_i\}$. By Lemma 2.1, Γ admits a Seifert surface system $\{F_i'\}$. The restriction of $\{F_i'\}$ to $E(V)$ gives a spanning surface system, say $\{F_i\}$, for $E(V)$ such that $(\{D_i\}, \{F_i\})$ is a Seifert surface system for ∂V. □

Let Γ be a g-handcuff graph shaped spine with a Seifert surface system $\{F_i\}$. We call the operation of (1) in Figure 2.2 a *band-crossing change* of $\{F_i\}$, and the operation of (2) in Figure 2.2 a *full-twist* of $\{F_i\}$. We remark that these operations can be obtained by a $1/\mathbb{Z}$-Dehn surgery along certain links in the complement of $\{F_i\}$ that are trivial in S^3 (see, for example, Figure 2.3).



(1) a band-crossing change (2) a full-twist

Figure 2.2. A band-crossing change and a full-twist of $\{F_i\}$.



(1) a band-crossing change (2) a full-twist

Figure 2.3. A band-crossing change and a full-twist of $\{F_i\}$ can be obtained by a $1/\mathbb{Z}$-Dehn surgery.

Lemma 2.3. *Any g-handcuff graph shaped spine Γ with a Seifert surface system $\{F_i\}$ can be unknotted by band-crossing changes and full-twists of $\{F_i\}$.*

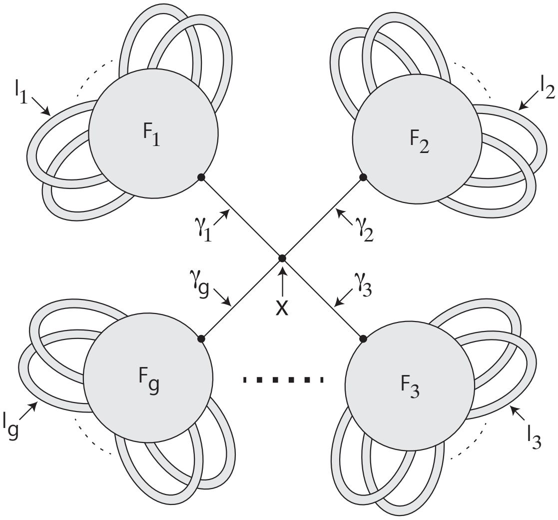


Figure 2.4. A g-handcuff graph shaped spine Γ with a Seifert surface system $\{F_i\}$, which is a "standard planar form".

Proof. We observe that Γ with $\{F_i\}$ can be transformed to a "standard planar form" (*cf*. [9]) by the following operations:

(1) a band-crossing change of $\{F_i\}$;
(2) a full twist of $\{F_i\}$;
(3) a crossing change between $\{F_i\}$ and $\{\gamma_i\}$;
(3) a crossing change among $\{\gamma_i\}$.

However, after some deformations of Γ with $\{F_i\}$, operations (3) and (4) can be exchanged with operation (1); see, for example, Figure 2.5.

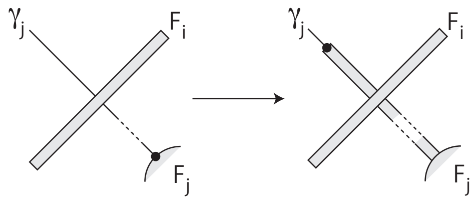


Figure 2.5. Operation (3) can be exchanged with operation (1).

If Γ with $\{F_i\}$ has a standard planar form, then Γ is unknotted and this completes the proof. We remark that in a standard planar form, $l_1 \cup \cdots \cup l_g$ is the trivial link. □

Lemma 2.4. *Let L be a reflexive link in S^3 which is contained in a compact* 3-*submanifold M in S^3. Suppose that L is null-homologous (respectively completely* *null-homologous) in M. Then the core link L^* in the 3-submanifold M' obtained by a $1/\mathbb{Z}$-Dehn surgery along L is also null-homologous (respectively completely null-homologous) in M'.*

Proof. Suppose that L is null-homologous (respectively completely null-homologous) in M. Then L bounds a Seifert surface F (respectively completely disjoint Seifert surface) in M. Defining $F^* = F \cap E(L)$, by a $1/\mathbb{Z}$-Dehn surgery, the meridian of the core link L^* intersects each component of ∂F^* at one point. This shows that F^* can be extended to a Seifert surface (respectively completely disjoint Seifert surface) for L^* in M'. Thus L^* is also null-homologous (respectively completely null-homologous) in M'. □

Lemma 2.5. *Let V be a handlebody in S^3. Then ∂V admits a (completely disjoint) Seifert surface system if and only if there exists a (completely) null-homologous link L in $E(V)$, which is reflexive in S^3, such that a handlebody can be obtained from $E(V)$ by a $1/\mathbb{Z}$-Dehn surgery along L.*

Proof. Suppose that there exists a (completely) null-homologous reflexive link L in $E(V)$ such that a handlebody, say W, can be obtained from $E(V)$ by a $1/\mathbb{Z}$-Dehn surgery along L. Then $V \cup W$ is a Heegaard splitting of S^3 and by Waldhausen's theorem [19], there exists a Seifert surface system $(\{D_i\}, \{D'_i\})$ for ∂W, where $\{D_i\}$ is a meridian disk system for V and $\{D'_i\}$ is a meridian disk system for W. Since L is (completely) null-homologous in $E(V)$, by Lemma 2.4, the core link L^* is also (completely) null-homologous in W. Therefore, we can obtain a (completely disjoint) Seifert surface system for $E(V)$ by tubing $\{D'_i\}$ along L^*.

Conversely, suppose that ∂V admits a (completely disjoint) Seifert surface system $(\{F_i\}, \{F'_i\})$, where $\{F_i\}$ and $\{F'_i\}$ are spanning surface systems for V and $E(V)$. By Remark 1.2, we may assume that each F_i is a disk. Take a regular neighborhood $N(F_i \cup \partial F'_i)$ in V and define $D_i = N(F_i \cup \partial F'_i) \cap (V - int\, N(F_i \cup \partial F'_i))$. Then D_i cuts off a solid torus $N(F_i \cup \partial F'_i)$ from V and thus $V - \bigcup int\, N(F_i \cup \partial F'_i)$ is a 3-ball. We can naturally take a g-handcuff graph shaped spine $\Gamma = l_1 \cup \cdots \cup l_g \cup \gamma_1 \cup \ldots \cup \gamma_g \cup x$ by using this decomposition of V; namely, l_i is a core loop of $N(F_i \cup \partial F'_i)$ intersecting F_i at one point, γ_i is dual to D_i and x is the point in the interior of $V - \bigcup int\, N(F_i \cup \partial F'_i)$. Since $\partial F'_i$ intersects ∂F_i at one point and is contained in the solid torus $N(F_i \cup \partial F'_i)$, F'_i can be extended to a (completely disjoint) Seifert surface system for Γ. By Lemma 2.3, Γ with $\{F'_i\}$ can be unknotted so that $E(V)$ becomes a handlebody, by band-crossing changes and full-twists of $\{F'_i\}$. These operations can be obtained by a $1/\mathbb{Z}$-Dehn surgery along a trivial link L in the complement of $\Gamma \cup \bigcup_i F'_i$. Since L is contained in $E(V) - \bigcup_i F'_i$, L is a (completely) null-homologous link in $E(V)$. Hence we obtain a (completely) null-homologous reflexive link L in $E(V)$ such that a handlebody can be obtained from $E(V)$ by a $1/\mathbb{Z}$-Dehn surgery along L. □

The following lemma will be used in the proof of Theorem 1.6, Step 3.

Lemma 2.6. *Let S be a Heegaard surface in S^3 which decomposes S^3 into two handlebodies V and V'. Let $\{D_i\}$ be a meridian disk system for V. Then there exist a null-homologous reflexive link L' in V', which yields a handlebody V'' by a $1/\mathbb{Z}$-Dehn surgery on L', and a meridian disk system $\{D_i''\}$ for V'' such that $(\{D_i\}, \{D_i''\})$ is a completely disjoint Seifert surface system for S in $V \cup V''$.*

Proof of Lemma 2.6. We take a g-handcuff graph shaped spine Γ of V with respect to D_i. Since Γ can be unknotted by crossing changes, there exists a null-homologous reflexive link L' in V' such that after a $1/\mathbb{Z}$-Dehn surgery along L', all loops of Γ bound mutually disjoint disks. Therefore, we obtain a handlebody V'' from V' by a $1/\mathbb{Z}$-Dehn surgery along L', and V'' admits a meridian disk system $\{D_i''\}$ so that $(\{D_i\}, \{D_i''\})$ is a completely disjoint Seifert surface system for S. □

Proof of Theorem 1.6.
(1) We prove the statement by induction on the genus $g(\partial M)$. Since the 3-sphere does not contain an incompressible closed surface, there exists a compressing disk D for ∂M in S^3. We divide the proof into two cases.

Case 1: $D \subset M$
Case 2: $D \subset E(M)$

In Case 1, define $M' = M - int\, N(D)$. By the inductive hypothesis, there exists a null-homologous reflexive link L' in M' such that handlebodies can be obtained from M' by a $1/\mathbb{Z}$-Dehn surgery along L'. This proves statement (1) of the theorem since M is obtained by adding a 1-handle $N(D)$ to M' in both cases where M' is connected and disconnected.

In Case 2, we take a maximal compression body W for ∂M in $E(M)$ [2]. If W is a handlebody (*i.e.*, $W = E(M)$), then the theorem follows from Lemma 2.2 and Lemma 2.5. Otherwise, since $g(\partial W) < g(\partial M)$, by the induction hypothesis, there exists a null-homologous reflexive link L' in each component of $E(M) - int\, W$ such that handlebodies can be obtained from the component by a $1/\mathbb{Z}$-Dehn surgery along L'. After these $1/\mathbb{Z}$-Dehn surgeries, $E(M)$ is a handlebody. Therefore, again by Lemma 2.2 and Lemma 2.5, there exists a null-homologous reflexive link L in M such that a handlebody can be obtained from M by a $1/\mathbb{Z}$-Dehn surgery along L. Finally, we recover the previous $1/\mathbb{Z}$-Dehn surgery on each component of $E(M) - int\, W$ to obtain the original $E(M)$.

(2) Suppose that there exists a completely null-homologous reflexive link L in M such that a handlebody can be obtained from M by a $1/\mathbb{Z}$-Dehn surgery along L. There exists a meridian disk system $\{D_i\}$ for the resultant handlebody. Since L is completely null-homologous in M, we can obtain a completely disjoint spanning surface system for M by tubing $\{D_i\}$ along L.

Conversely, suppose that M admits a completely disjoint spanning surface system $\{F_i\}$. In the following 3 steps, we convert M and $E(M)$ into two handlebodies V and V'' so that (V, V'') admits a meridian disk system $(\{D_i\}, \{D_i''\})$ with $\partial F_i = \partial D_i$.

Step 1. By (1) of this theorem, there exists a null-homologous link reflexive L in $E(M)$ such that a handlebody can be obtained from $E(M)$ by a $1/\mathbb{Z}$-Dehn surgery along L. Let V' be the resultant handlebody obtained from $E(M)$ and note that $M \cup V'$ is again the 3-sphere.

Step 2. We note that there exists a degree one map from M to a handlebody V which sends each F_i to a meridian disk D_i of V and preserves the boundary of M (*cf.* [10, Theorem 2], [5, Theorem 5]). We naturally extend this degree one map to a degree one map $\phi : S^3 = M \cup V' \to X = V \cup V'$ as follows:

(1) V' is contained in X by an inclusion;
(2) each F_i is sent to a meridian disk D_i of the handlebody $\phi(M) = V$;
(3) the remnant $M - \bigcup int\ N(F_i)$ is sent to the 3-ball $V - \bigcup int\ N(D_i)$.

Since $\phi_* : \pi_1(S^3) \to \pi_1(X)$ is surjective [6, Lemma 15.12], X is homeomorphic to S^3 [13,15,16].

Step 3. By Lemma 2.6, there exists a null-homologous reflexive link L' in V' and a meridian disk system $\{D_i''\}$ for a handlebody V'' obtained from V' by a $1/\mathbb{Z}$-Dehn surgery along L' such that $(\{D_i\}, \{D_i''\})$ is a completely disjoint Seifert surface system for (V, V'').

Since the degree one map ϕ is a boundary preserving map by condition (1), $(\{F_i\}, \{D_i''\})$ is a completely disjoint Seifert surface system for (M, V''). By Lemma 2.5, there exists a completely null-homologous reflexive link L_0 in M such that a handlebody can be obtained from M by a $1/\mathbb{Z}$-Dehn surgery along L_0. Moreover, by the proof of Lemma 2.5, we can take L_0 so that $L_0 \cap \bigcup F_i = \emptyset$. Thus the completely disjoint spanning surface system $\{F_i\}$ is contained in the resultant handlebody V_0 obtained from M by a $1/\mathbb{Z}$-Dehn surgery along L_0. □

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