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Locating the boundary peaks of least-energy solutions to a singularly perturbed Dirichlet problem

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Abstract. We consider the problem

$$\varepsilon^2 \Delta v - v - \gamma_1 V v + f(v) = 0 \quad \Delta V + \gamma_2 |v|^2 = 0, \quad v = V = 0 \text{ on } \partial\Omega,$$

where $\Omega \subset \mathbb{R}^3$ is a smooth and bounded domain, $\varepsilon,\ \gamma_1,\ \gamma_2 > 0,\ v,\ V : \Omega \to \mathbb{R}$, $f : \mathbb{R} \to \mathbb{R}$. We prove that this system has a *least-energy solution* v_ε which develops, as $\varepsilon \to 0^+$, a single spike layer located near the boundary, in striking contrast with the result in [37] for the single Schrödinger equation. Moreover the unique peak approaches the *most curved* part of $\partial\Omega$, *i.e.*, where the boundary mean curvature assumes its maximum. Thus this elliptic system, even though it is a Dirichlet problem, acts more like a Neumann problem for the single-equation case. The technique employed is based on the so-called energy method, which consists in the derivation of an asymptotic expansion for the energy of the solutions in powers of ε up to sixth order; from the analysis of the main terms of the energy expansion we derive the location of the peak in Ω.

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