

## On naturally reductive left-invariant metrics of $SL(2, \mathbb{R})$

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**Abstract.** On any real semisimple Lie group we consider a one-parameter family of left-invariant naturally reductive metrics. Their geodesic flow in terms of Killing curves, the Levi Civita connection and the main curvature properties are explicitly computed. Furthermore we present a group theoretical revisit of a classical realization of all simply connected 3-dimensional manifolds with a transitive group of isometries due to L. Bianchi and É. Cartan. As a consequence one obtains a characterization of all naturally reductive left-invariant Riemannian metrics of  $SL(2, \mathbb{R})$ .

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