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Generic cycles, Lefschetz representations, and the generalized Hodge and Bloch conjectures for Abelian varieties

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Abstract. We prove Bloch's conjecture for correspondences on powers of complex Abelian varieties, that are "generically defined". As an application we establish vanishing results for (skew-)symmetric cycles on powers of Abelian varieties and we address a question of Voisin concerning (skew-)symmetric cycles on powers of K3 surfaces in the case of Kummer surfaces. We also prove Bloch's conjecture in the following situation. Let γ be a correspondence between two Abelian varieties A and B that can be written as a linear combination of products of symmetric divisors. Assume that A is isogenous to the product of an Abelian variety of totally real type with the power of an Abelian surface. We show that γ satisfies the conclusion of Bloch's conjecture. A key ingredient consists in establishing a strong form of the generalized Hodge conjecture for Hodge sub-structures of the cohomology of A that arise as sub-representations of the Lefschetz group of A. As a by-product of our method, we use a strong form of the generalized Hodge conjecture established for powers of Abelian surfaces to show that every finite-order symplectic automorphism of a generalized Kummer variety acts as the identity on the zero-cycles.

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