An equivalence principle between polynomial and simultaneous Diophantine approximation

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Abstract. Mahler partitioned the real numbers into S, T and U-numbers subject to the growth of the sequence of Diophantine exponents $(w_n(\zeta))_{n\geq 1}$ associated to a real number ζ. Koksma introduced a similar classification that turned out to be equivalent. We add two more equivalent definitions in terms of classical exponents of Diophantine approximation. One concerns certain natural assumptions on the decay of the sequence $(\lambda_n(\zeta))_{n\geq 1}$ related to simultaneous rational approximation to $(\zeta, \zeta^2, \ldots, \zeta^n)$. Thereby we obtain a much clearer picture on simultaneous approximation to successive powers of a real number in general. The other variant of Mahler's classification deals with uniform approximation by algebraic numbers. We further provide various other applications of our underlying method to exponents of Diophantine approximation and metric theory.

Mathematics Subject Classification (2010): 11J13 (primary); 11J82, 11J83 (secondary).