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The connecting solution of the Painlevé phase transition model

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Abstract. The second Painlevé O.D.E. $y'' - xy - 2y^3 = 0$, $x \in \mathbb{R}$, is known to play an important role in the theory of integrable systems, random matrices, Bose-Einstein condensates and other problems. The generalized second Painlevé equation $\Delta y - x_1 y - 2y^3 = 0$, $(x_1, x_2) \in \mathbb{R}^2$, is obtained by multiplying by $-x_1$ the linear term u of the Allen-Cahn equation $\Delta u = u^3 - u$. It involves a non autonomous potential $H(x_1, y)$ which is bistable for every fixed $x_1 < 0$, and thus describes as the Allen-Cahn equation a phase transition model. The scope of this paper is to construct a solution y connecting along the vertical direction x_2, the two branches of minima of H parametrized by x_1. This solution plays a similar role that the heteroclinic orbit for the Allen-Cahn equation. It is the the first to our knowledge solution of the Painlevé P.D.E. both relevant from the applications point of view (liquid crystals), and mathematically interesting.

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